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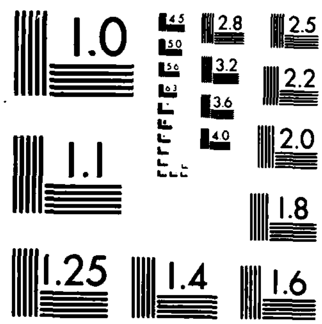

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CONTRACTOR REPORT

THROUGH-FLOW MODELS FOR
MASS AND MOMENTUM AVERAGED
VARIABLES

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ABSTRACT

The turbomachinery through-flow equations are reformulated for mass and momentum averaged quantities. The background of this analysis is the need for an improved assessment of the accuracy of through-flow computations. Traditional through-flow analyses are based on density weighted averaged quantities reducing to an area average in incompressible flows. On the other hand, experimental data are usually evaluated under the form of mass-averaged quantities, particularly with regard to the overall energy balance and efficiency estimations. The transition between these two sets of quantities is usually taken into account by introducing an averaged aerodynamic blockage factor in addition to the blade blockage factor resulting from the density averaged quantities.

The present analysis provides a rigorous derivation for the momentum averaged flow quantities and shows that some strong assumptions on the nature of the non-axisymmetric flow components are necessary in order to justify the current practice of introduction of aerodynamic blockage.

The recent availability of detailed flow data in single and two stage axial compressors allows a partial validation of these assumptions, by the comparison of the various non-axisymmetric components. In addition some guidelines are provided relating the blockage factors to wake and loss coefficients.

INTRODUCTION

Classical through-flow analysis models, as applied in aerodynamic design systems of turbomachinery are generally based on density weighted averaged quantities (reducing to area averages in incompressible flows), Smith (1966), Hirsch and Warzee (1976), (1979), Jennions and Stow (1984). This formulation leads to the introduction of additional "interaction" terms, having the same structure as the turbulent Reynolds stress, but arising from the non-axisymmetry of the flow and contributing to the averaged radial equilibrium. These interaction terms have to be evaluated explicitly in quasi-three dimensional modifications of turbomachinery flows whereby iterative computations of through-flows (S2 surfaces) and axisymmetric blade to blade (S1 surfaces) flows are performed, with appropriate data being transmitted from one family to the other family of surfaces, Hirsch and Warzee (1979), Jennions and Stow (1985).

An alternative to the explicit evaluation of the interaction terms as describing the effects of the non-axisymmetry on the averaged flow, is to introduce instead, an aerodynamic blockage factor, based on mass flow considerations, in addition to the blade blockage resulting from the density weighted averaging. This approach is followed by Calvert and Ginder (1985) in order to define a consistent quasi-3D interactive procedure. These authors rightly point out, that their aerodynamic blockage factor should contain the same information on the non-axisymmetry of the flow as the interaction terms, and that its use in the continuity equation replaces the interaction terms in the radial equilibrium momentum equation. As a consequence of this, Calvert and Ginder define a mass-averaged through-flow instead of the density weighted area averaged flow considered by the previously mentioned authors.

Earlier, both present authors had separately stressed the importance of the introduction of an aerodynamic blockage in through-flow evaluation methods, Hirsch & Denton (1981), Dring (1984). More particularly, this last reference proposes a quantitative definition of blockage as the ratio between mass averaged and area-averaged axial velocity components. Based on the extensive data base for single and multistage axial compressors obtained in the last years at United Technology by the second author and his coworkers, Dring et al. (1979), (1982), (1983), Wagner et al. (1983), (1984a), (1984b), quantitative evaluations of the blockage factor were made possible, showing its importance particularly near the end walls. This has been confirmed more recently by Dring and Joslyn (1985), who showed that both the level and spanwise distribution of aerodynamic blockage are important and have a strong impact on the computed flow field at the outlet of blade rows. In this latter analysis the computed quantities were also considered as mass averaged quantities.

The debate between the two families of averaged quantities is central to the validation of through-flow models. On one hand, coherent through-flow models can be defined for density-area averaged quantities, but on the other hand physical arguments and the strong connection between mass averaged quantities, like stagnation pressure, total energy and machine efficiency, are essential to the correct estimation of the energy exchange within the turbomachinery blade row.

The present report aims at the derivation of a consistent through-flow model for mass, or more precisely, momentum averaged flow variables . A consistent model can be obtained, at the cost of six different blockage coefficients, depending on which components of the momentum flux are to be considered.

As will be shown, if the strong assumption is made of the equality of all the blockage coefficients, then a simplified model is obtained, which entirely justifies the semi-intuitive approaches followed by Calvert and Ginder (1985) and Dring and Joslyn (1985).

Comparison with experimental data allows an evaluation of the limits of validity of this assumption and guidelines are presented for the relation of the aerodynamic blockage factors with loss coefficients in order to enable its introduction in design systems.

Section one will present the recently derived averaged form of the conservation equations in vector form, from which different formulations can be obtained.

Section two will discuss the important energy conservation equation and the impact various definitions might have on a through-flow analysis model.

The momentum averaged equations are derived in section three and the influence of various blockage coefficients is investigated.

1. DEFINITION OF PASSAGE AVERAGED FLOW EQUATIONS

All flow equations are averaged over the blade passage, defined as the region between the suction surface $\theta = \theta_s$ and the pressure surface of the next blade, $\theta = \theta_p$, figure 1.1. The area average of an arbitrary quantity is defined as

$$\bar{A} = \frac{1}{\theta_p - \theta_s} \int_{\theta_p}^{\theta_s} A \, d\theta \tag{1.1}$$

With the introduction of the blade thickness, d, in the tangential direction, and the blade pitch, s, one can write $\bar{A}$ as

$$\bar{A} = \frac{1}{\frac{2\pi b}{N}} \int_{p}^{s} A \, d\theta \tag{1.2}$$

where

$$b = 1 - \frac{d}{s} \tag{1.3}$$

and N is the number of blades.

This averaging procedure is applied to all the conservat n equations (mass, momentum and energy) and in order to handle compressibility effects a density weighted passage average is defined, Hirsch and Warzee (1979), by

$$\bar{\rho}\tilde{A} = \overline{\rho A} = \frac{1}{2\pi b/N} \int_p^s \rho A \, d\theta \tag{1.4}$$

The deviations from axisymmetry are defined by A' and A" according to

$$A = \bar{A} + A' = \tilde{A} + A'' \tag{1.5}$$

with

$$\overline{A'} = \overline{\rho A''} = 0 \tag{1.6}$$

In the following we will use the notation $\bar{A}^{(a)}$ instead of $\tilde{A}$, in order to distinguish those quantities from corresponding mass-averaged values $\bar{\bar{A}}^{(m)}$.

1.1 Turbomachinery Flow Model

It is important, in an attempt to assess the validity of different assumptions on through-flow quantities, to keep in mind the approximations at the basis of the flow models generally used in turbomachinery.

The essential approximation is expressed by a distributed loss model, Hirsch (1985), in which the shear stresses are replaced by a loss (entropy) generating friction force $\vec{F}_f$, considered as a distributed force, defined by the total pressure loss coefficients. Similarly, the energy and entropy equations are simplified by the assumption that the shear stress work is exactly balanced by the heat conduction effects, leading to the following set of equations, written in the relative system

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}(\rho\vec{w}) = 0$$

$$\frac{\partial(\rho\vec{w})}{\partial t} + \vec{\nabla}(\rho\vec{w} \otimes \vec{w}) = -\vec{\nabla}p - 2\rho(\vec{\omega}\times\vec{w}) + \rho\omega^2\vec{r} + \rho\vec{F}_f$$

$$\frac{\partial(\rho I)}{\partial t} + \vec{\nabla}(\rho\vec{w}I) = \frac{\partial p}{\partial t} \tag{1.7}$$

$$\frac{\partial}{\partial t}(\rho s) + \vec{\nabla}(\rho\vec{w}s) = \frac{w}{T}\,\rho\,F_f$$

In these equations, written in conservative form, $\vec{w}$ is the relative velocity related to the absolute velocity $\vec{v}$ by

$$\vec{v} = \vec{w} + \vec{u} = \vec{w} + \vec{\omega}\times\vec{r} \tag{1.8}$$

for a steady rotation of angular velocity $\vec{\omega}$.

The enthalpy, I, is defined by

$$I = h + \frac{\vec{w}^2}{2} - \frac{\vec{u}^2}{2} = H - \vec{u}\cdot\vec{v} \tag{1.9}$$

where the stagnation enthalpy, H, is

$$H = h + \frac{\vec{v}^2}{2} \tag{1.10}$$

These equations are considered as Reynolds-averaged for the turbulence fluctuations, and we will assume that they are also averaged for the unsteady effects, leaving us with a steady flow in the relative system.

An interesting discussion of the interrelation between different averaging scales in time can be found in Adamczyk (1984).

For steady relative flow, we have the simplified flow model

$$\left\{ \begin{aligned} &\vec{\nabla}(\rho\vec{w}) = 0 \\ &\vec{\nabla}(\rho\vec{w} \otimes \vec{w}) = -\vec{\nabla}p - 2\rho(\vec{\omega} \times \vec{w}) + \rho\omega^2\vec{r} + \rho\vec{F}_f \\ &\vec{\nabla}(\rho\vec{w}I) = 0 \\ &\vec{\nabla}(\rho\vec{w}s) = \frac{w}{T}\rho\ F_f \end{aligned} \right. \tag{1.11}$$

An alternate formulation is provided by Crocco's form for the momentum equation, where the pressure term is eliminated by application of the entropy relation

$$T\,ds = dh - \frac{dp}{\rho} \tag{1.12}$$

coupled to the non-conservative forms for the energy and entropy equations by applying the continuity equation. This leads to

$$
\begin{cases}
\vec{\nabla}(\rho\vec{w}) = 0 \\
-\vec{w}\times\vec{\zeta} = T\vec{\nabla}s - \vec{\nabla}I + \vec{F}_f \\
(\vec{w}\vec{\nabla})I = 0 \\
T(\vec{w}\vec{\nabla})s = w\,F_f
\end{cases}
\tag{1.13}
$$

It is important to remember that the entropy equation is not independent of the other equations. Hence, when the entropy equation is applied, as is the case in through-flow models, one of the momentum equations has to be discarded.

In equation (1.13), the absolute vorticity $\vec{\zeta}$ has been introduced, defined by

$$\vec{\zeta} = \vec{\nabla}\times\vec{v} \tag{1.14}$$

1.2 Passage Averaged Equations

The above system of equations are passage averaged, following the definition (1.2). A detailed derivation can be found in Hirsch (1984). As is well known, the averaging procedure introduces a blockage factor, b, and a body force, $\vec{f}_B$, as a consequence of the three-dimensionality of the flow. The following system is obtained:

Continuity equation

$$\vec{\nabla}(\overline{\rho\,\vec{w}}b) = 0 \tag{1.15}$$

Momentum equation

$$\vec{\nabla}(\overline{\rho\vec{w} \otimes \vec{w}b}) = -b\vec{\nabla}\bar{p} + \bar{\rho}\,\omega^2\,\vec{r}b - 2\overline{\rho(\vec{\omega} \times \vec{w})b}$$

$$+ \overline{\rho\vec{F}_f}\,b + \bar{\rho}\,\vec{f}_B\,b \qquad (1.16)$$

Energy equation

$$\vec{\nabla}(\overline{\rho\,\vec{w}\,I\,b}) = 0 \qquad (1.17)$$

The blockage factor, b, is defined by equation (1.3) and the blade force is equal to

$$\bar{\rho}\,\vec{f}_B = \frac{r}{bs}(p_p\,\vec{n}^{(p)} - p_p\vec{n}^{(s)}) - (\bar{p} - \frac{p_p + p_s}{2})\frac{\vec{\nabla}b}{b} \qquad (1.18)$$

where $\vec{n}^{(p)}$ and $\vec{n}^{(s)}$ are the normals to the pressure and suction surface, respectively, with components in cylindrical coordinates, for instance, for the pressure side

$$\begin{cases} -n_r^{(p)} = \dfrac{\partial\theta_p}{\partial r} = \dfrac{1}{r}\tan\varepsilon'_p \\ -n_z^{(z)} = \dfrac{\partial\theta_p}{\partial z} = \dfrac{1}{r}\tan\beta'_p \\ n_\theta^{(p)} = 1 \end{cases} \qquad (1.19)$$

The lean angle ε'_p and the blade angle β'_p of the pressure side are hereby introduced with similar relations defined for the suction side. The entropy equation will be treated separately.

In cylindrical coordinates (r,θ,z) one obtains the following projections,

Continuity equation

$$\frac{\partial}{\partial r}(\overline{\rho w_r}\, b\, r) + \frac{\partial}{\partial z}(\overline{\rho w_z}\, b\, r) = 0 \tag{1.20}$$

Momentum equations

$$\frac{1}{br}\frac{\partial}{\partial r}(\overline{\rho w_r\, w_r}\, b\, r) + \frac{1}{br}\frac{\partial}{\partial z}(\overline{\rho w_r\, w_z}\, b\, r) - \frac{\overline{\rho w_\theta w_\theta}}{r}$$

$$= -\frac{\partial\overline{p}}{\partial r} + \overline{\rho}\,\omega\, r^2 + 2\overline{\rho w_\theta}\omega + \overline{\rho}(F_{fr} + f_{Br})$$

$$\frac{1}{br}\frac{\partial}{\partial r}(\overline{\rho w_r w_z} br) + \frac{1}{br}\frac{\partial}{\partial z}(\overline{\rho w_z w_z} br) = -\frac{\partial\overline{p}}{\partial z} + \overline{\rho}(F_{fz} + f_{Bz}) \tag{1.21}$$

$$\frac{1}{br}\frac{\partial}{\partial r}(\overline{\rho w_r w_\theta} br) + \frac{1}{br}\frac{\partial}{\partial z}(\overline{\rho w_z w_\theta} br) + \frac{\overline{\rho w_\theta w_r}}{r}$$

$$= -2\overline{\rho w_r}\omega + \overline{\rho}(F_{f\theta} + f_{B\theta})$$

Energy equation

$$\frac{\partial}{\partial r}(\overline{\rho w_r I} br) + \frac{\partial}{\partial z}(\overline{\rho w_z I} br) = 0 \tag{1.22}$$

An interesting alternative formulation for the mass and energy conservation laws is obtained in the axisymmetric coordinate system (m,n,θ), figure 1-2, where n is normal to the meridional streamline, m.

Expressing mass conservation for the streamtube of thickness B_1, one has, instead of equation (1.20), since $w_n = 0$ and assuming that the shape of the meridional streamline is nearly axisymmetric,

$$\frac{\partial}{\partial m}(\overline{\rho w_m A}) = 0 \tag{1.23}$$

where $A = 2\pi r b B_1$, and for the energy equation

$$\frac{\partial}{\partial m}(\overline{\rho w_m I A}) = 0 \tag{1.24}$$

In the above expressions, no decision has been made with regard to the nature of the averaged variables to be considered. The current option to be found in the literature is the density-weighted, area average, according to the definition (1.4), Hirsch and Warzee (1979), Jennions and Stow (1984). Writing the density weighted, area averaged quantities with a superscript (a), instead of the tilda in equation (1.4), one obtains the following consistent through-flow model.

$$\left\{\begin{aligned} &\frac{\partial}{\partial r}(\bar{\rho} b r \bar{w}_r^{(a)}) + \frac{\partial}{\partial z}(\bar{\rho} b r \bar{w}_z^{(a)}) = 0 \\ &\vec{\nabla}(\overline{\rho \vec{w}}^{(a)} \otimes \vec{w}^{(a)} b r) = -b\vec{\nabla}\bar{p} + \bar{\rho} b \vec{\bar{F}} + \vec{\nabla}(\bar{\bar{\tau}}^{(s)} b) \\ &\vec{\nabla}(\overline{\rho \vec{w}}^{(a)} \bar{I}^{(a)} b) = -\vec{\nabla}(\overline{\rho \vec{w}'' I'' b}) \end{aligned}\right. \tag{1.25}$$

In equations (1.25), the term $\vec{\bar{F}}$ is the sum of all the forces,

$$\vec{\bar{F}} = \omega^2 \vec{r} - 2\bar{\rho}(\omega \times \vec{\bar{w}}^{(a)}) + \vec{F}_f + \vec{f}_B \tag{1.26}$$

and the additional stress term $\bar{\bar{\tau}}^{(s)}$ represents the sum of the "interaction" terms expressed as the gradient of a "secondary" stress factor

$$\bar{\bar{\tau}}^{(s)} = - \overline{\rho \vec{w}'' \otimes \vec{w}''} \tag{1.27}$$

The energy equation shows that the averaged total energy, $\bar{I}^{(a)}$, defined by

$$\bar{I}^{(a)} = \bar{h}^{(a)} + \frac{\overline{\vec{w}^2}^{(a)}}{2} - \frac{\vec{u}^2}{2} \tag{1.28}$$

is not constant along a density area averaged streamline. This has important consequences on the consistency of through-flow models and will be discussed in more detail in section 2.1.

1.3 Averaged Crocco's Form of the Momentum Equations

By averaging the entropy relation (1.12), one can replace the pressure gradient and the blade force, $\vec{f}_B$, by averaged thermodynamic variables $\bar{h}^{(a)}$ and $\bar{T}^{(a)}$ and by an alternate blade force function of the enthalpy variations between pressure and suction side, Hirsch and Warzee (1976), Hirsch (1984).

The details of the calculations can be found in these references, where the simplifying assumption of an axisymmetric entropy has been made, in accordance with the fact that the friction force $\vec{F}_f$ is considered, in practical calculations as an axisymmetric quantity. Note however that this assumption can easily be removed as seen in section 1 of Hirsch (1984). One obtains, for steady state conditions, with the relation

$$\vec{\nabla}\overline{p} - \overline{\rho\vec{f}_B} = \overline{\rho\vec{\nabla}h}^{(a)} - \overline{\rho}\,\overline{T}^{(a)}\,\vec{\nabla}s - \overline{\rho}\,\vec{f}_h \qquad (1.29)$$

where $\vec{f}_h$ is a body force term expressed as a function of h,

$$\overline{\rho\vec{f}_h} = -\frac{1}{b}[(\rho h''\vec{n})_p - (\rho h''\vec{n})_s] + \overline{h''\vec{\nabla}\rho} \qquad (1.30)$$

the following expression for equation (1.16)

$$\frac{1}{b}\vec{\nabla}(\overline{\rho\vec{w} \otimes \vec{w}}b) = \overline{\rho}\,\overline{T}^{(a)}\,\vec{\nabla}s - \overline{\rho\vec{\nabla}h}^{(a)} + \overline{\rho\vec{F}_h} \qquad (1.31)$$

with

$$\overline{\rho\vec{F}_h} = \overline{\rho}\omega^2\vec{r} - 2\overline{\rho(\vec{\omega}\times\vec{w})} + \overline{\rho\vec{F}_f} + \overline{\rho\vec{f}_h} \qquad (1.32)$$

When introduced in the density weighted area averaged momentum equation (1.25) one obtains, with the application of the continuity equation

$$-\vec{\overline{w}}^{(a)} \times \vec{\overline{\zeta}}^{(a)} = \overline{T}^{(a)}\vec{\nabla}s - \vec{\nabla}\hat{I}^{(a)} + \vec{F}_f + \vec{f}_h + \frac{1}{b\overline{\rho}}\vec{\nabla}(\overline{\overline{\tau}}^{(s)}b) \qquad (1.33)$$

In equation (1.33), the averaged absolute vorticity is introduced as

$$\vec{\bar{\zeta}}^{(a)} = \vec{\nabla} \times \vec{\bar{v}}^{(a)}$$

and the total energy of the density weighted averaged flow, $\hat{I}^{(a)}$

$$\hat{I}^{(a)} = \bar{h}^{(a)} + \frac{\vec{\bar{w}}^{(a)\,2}}{2} - \frac{\vec{u}^{2}}{2}$$

$$= \bar{I}^{(a)} - \frac{\overline{\rho \vec{w}'' \vec{w}''}}{2\bar{\rho}} \equiv \bar{I}^{(a)} - \bar{k}^{(a)} \qquad (1.34)$$

The quantity $\bar{I}^{(a)}$ is the averaged total energy of the flow, while $\hat{I}^{(a)}$ is the total energy of the averaged flow. These two quantities differ by the average of the kinetic energy of the fluctuations $\vec{w}''$.

2. THE AVERAGED ENERGY EQUATION

The total energy I (or H) plays an essential role in all the through-flow models, since the through-flow computations have to rely on the energy relation along the meridional streamlines. In most, if not all, of the through-flow programs based on axisymmetric models, the constancy of the rothalpy is applied in order to relate quantities in two consecutive calculation stations. When dealing with the influence of the non-axisymmetric effects it is essential to be able to estimate their influence on the energy transport and exchange.

From the third of equations (1.25), one can see that neither the density averaged total energy $\overline{I}^{(a)}$, which satisfies

$$\overline{\rho}\,\overline{w}_m^{(a)}\ \frac{\partial}{\partial m}\ \overline{I}^{(a)} \;=\; -\frac{1}{b}\vec{\nabla}\,\overline{(\rho\vec{w}''I''b)} \tag{2.1}$$

nor the total energy of the average flow $\hat{I}^{(a)}$, which obeys the equation

$$\overline{\rho}\,\overline{w}_m^{(a)}\ \frac{\partial}{\partial m}\ \hat{I}^{(a)} \;=\; -\frac{1}{b}\vec{\nabla}\,\overline{(\rho\vec{w}''I''b)} - \frac{1}{b}\vec{\nabla}(\overline{\rho\vec{w}}^{(a)}\overline{k}^{(a)}b)$$

$$= -\frac{1}{b}\vec{\nabla}\,\overline{(\rho\vec{w}''I''b)} - \overline{\rho}\,\overline{w}_m^{(a)}\ \frac{\partial}{\partial m}\ \overline{k}^{(a)} \tag{2.2}$$

are conserved along a streamline $\overline{w}_m^{(a)}$. *On the other hand, if* the streamsurfaces are assumed to be axisymmetric, a unique mass averaged total energy can be defined by

$$\overline{\rho\vec{w}I} \;\equiv\; \overline{\rho}\,\overline{\vec{w}}^{(a)}\,\overline{\overline{I}}^{(m)} \;=\; \frac{1}{b}\int_p^s (\rho\vec{w}I)\,d\theta \tag{2.3}$$

from equation (1.14)

$$\nabla(\overline{\rho}\,\overline{\vec{w}}^{(a)}\,\overline{\overline{I}}^{(m)}b) \;=\; 0 \tag{2.4}$$

or taking into account the continuity equation (1.15)

$$\overline{\rho}\,\overline{\vec{w}}^{(a)}\,\vec{\nabla}\overline{\overline{I}}^{(m)} \;=\; 0 \tag{2.5}$$

This shows that the mass-averaged enthalpy is conserved along a streamline defined by the density weighted, area average flow (which actually is the mass-flow conserving average). Indeed, equation (2.5) can be written as

$$\bar{\rho}\, \bar{w}_m^{(a)} \frac{\partial}{\partial m} \bar{\bar{I}}^{(m)} = 0 \tag{2.6}$$

A total energy of the mass averaged flow is defined from

$$\begin{aligned}
\bar{\bar{I}}^{(m)} &= \bar{\bar{h}}^{(m)} + \frac{\overline{\vec{w}^2}^{(m)}}{2} - \frac{\vec{u}^2}{2} \\
&= \bar{\bar{h}}^{(m)} + (\bar{\bar{\vec{w}}}^{(m)})^2 \frac{1}{2} - \frac{\vec{u}^2}{2} + \frac{\overline{\rho w_m \vec{w}''' \vec{w}'''}}{2\bar{\rho}\, \bar{w}_m^{(a)}} \\
&\equiv \hat{I}^{(m)} + \bar{\bar{k}}^{(m)}
\end{aligned} \tag{2.7}$$

where the mass averaged velocity vector is defined by

$$\bar{\bar{\vec{w}}}^{(m)} \equiv \frac{\overline{\rho w_m \vec{w}}}{\bar{\rho}\, \bar{w}_m^{(a)}} \tag{2.8}$$

and the fluctuations $\vec{w}'''$ are determined by

$$\vec{w} = \bar{\bar{\vec{w}}}^{(m)} + \vec{w}''' \tag{2.9}$$

with

$$\overline{\rho w_m \vec{w}'''} = 0 \tag{2.10}$$

Inserting (2.7) into (2.6), one obtains

$$\bar{\rho}\, \bar{w}_m(a)\, \frac{\partial}{\partial m}\, \hat{I}^{(m)} \;=\; -\, \bar{\rho}\, \bar{w}_m^{(a)}\, \frac{\partial}{\partial m}\, \bar{\bar{k}}^{(m)} \qquad (2.11)$$

showing that $\hat{I}^{(m)}$, the total energy of the mass averaged flow is not rigorously conserved along a density area averaged streamline.

According to Jennions and Stow (1985), the contribution of the kinetic energy of the fluctuations, can be as high as 30% of the total energy of the averaged mean flow.

<u>Non-axisymmetric streamsurfaces - Radial mixing effects</u>

If the blade to blade streamsurfaces are not close to an axisymmetric shape, as a consequence of secondary flows, the assumption (2.3) ceases to be strictly valid.

Instead, the mean values appearing in equation (1.22) are evaluated as follows, writing, see figure 1.2,

$$\begin{aligned} w_r &= w_m \sin\sigma = w_m \sin(\bar{\sigma} + \sigma') \\ w_z &= w_m \cos\sigma = w_m \cos(\bar{\sigma} + \sigma') \end{aligned} \qquad (2.12)$$

introducing hereby the averaged streamsurface slope angle $\bar{\sigma}$, and the "twist" angle σ' defined as the difference between the actual angle σ and the mean (axisymmetric) value $\bar{\sigma}$

$$\sigma(r,\theta,z) = \bar{\sigma}(r,z) + \sigma'(r,\theta,z) \qquad (2.13)$$

Decomposing (2.12) and assuming that

$$\overline{\cos\sigma'} = 1 \quad \text{and} \quad \overline{\sin\sigma'} = 0$$

one can estimate the averaged products as follows. For the first term, one would have

$$\overline{\rho\, w_m I \sin(\bar{\sigma}+\sigma')} \cong \overline{\rho\, w_m I \sin\bar{\sigma}} + \overline{\rho\, w_m I \sin\sigma'} \cdot \cos\bar{\sigma} \qquad (2.14a)$$

and for the second term

$$\overline{\rho\, w_m I \cos(\bar{\sigma}+\sigma')} \cong \overline{\rho\, w_m I \cos\bar{\sigma}} - \overline{\rho\, w_m I \sin\sigma'} \cdot \sin\bar{\sigma} \qquad (2.14b)$$

These expressions are introduced in the energy equation (1.22). The first terms of (2.14) give the axisymmetric contribution of the l.h.s. of equation (2.4). Indeed, defining $\bar{\bar{I}}^{(m)}$ by

$$\overline{\rho\, w_m I} = \bar{\rho}\, \bar{w}_m^{(a)}\, \bar{\bar{I}}^{(m)} \qquad (2.15)$$

one has

$$\frac{\partial}{\partial r}(\overline{\rho\, b r\, w_r I}) + \frac{\partial}{\partial r}(\overline{\rho\, b r\, w_z I}) = \frac{\partial}{\partial r}(\bar{\rho}\, b r\, \bar{w}_m^{(a)} \sin\bar{\sigma}\bar{\bar{I}}^{(m)})$$

$$+ \frac{\partial}{\partial z}(\bar{\rho}\, b r\, \bar{w}_m^{(a)} \cos\bar{\sigma}\, \bar{\bar{I}}^{(m)}) + \frac{\partial}{\partial r}(b\, r\overline{\rho\, w_m I \sin\sigma'} \cdot \cos\bar{\sigma})$$

$$- \frac{\partial}{\partial z}(b\, r\overline{\rho\, w_m I \sin\sigma'} \cdot \sin\bar{\sigma}) = 0 \qquad (2.16)$$

Note that the definition (2.15) of $\overline{\overline{I}}^{(m)}$ is identical to the definition (2.3) in the case of axisymmetry.

The first two terms reduce to the left hand side of (2.6) and (2.15) becomes, with

$$\overline{\rho\, w_m I \sin\sigma'} = \overline{(\rho w_r)' I''} = \overline{\rho' w_r' I''} \qquad (2.17)$$

where the energy fluctuations I"' are defined by

$$I = \overline{\overline{I}}^{(m)} + I'' \qquad (2.18)$$

$$\bar{\rho}\, b r\, \bar{w}_m^{(a)} \frac{\partial}{\partial m} \overline{\overline{I}}^{(m)} = \frac{\partial}{\partial r}(b\, r \overline{\rho' w_r' I''} \cos\bar{\sigma}) - \frac{\partial}{\partial z}(b\, r \overline{\rho' w_r' I''} \sin\bar{\sigma})$$

$$= \frac{\partial}{\partial n}(b\, r \overline{\rho' w_r' I''}) + \frac{b\, r \overline{\rho' w_r' I''}}{R_m} \qquad (2.19)$$

where R_m is the radius of curvature of the average streamline m, figure 1.2. The derivatives in the direction normal to the axisymmetric (averaged) streamsurface $\frac{\partial}{\partial n}$ appears defined by

$$\frac{\partial}{\partial n} = \cos\bar{\sigma} \frac{\partial}{\partial r} - \sin\bar{\sigma} \frac{\partial}{\partial z}$$

and the curvature is defined by

$$\frac{1}{R_m} = - \frac{\partial \bar{\sigma}}{\partial m}$$

The last term of equation (2.19) can be neglected since the <u>gradients</u> of the non-axisymmetric contributions are much more significant than their amplitude. Therefore, the non-axisymmetric energy equation, generalizing equation (2.6) becomes

$$\bar{\rho}\, b\, r\, \bar{w}_m^{(a)} \frac{\partial \bar{\bar{I}}^{(m)}}{\partial m} = \frac{\partial}{\partial n}(b\, r\, \overline{\rho'\, w_r'\, I''}) \qquad (2.20)$$

Note that the right hand side of equation (2.20) represents a source term originating from the radial component of the secondary velocity field and describes therefore a <u>radial mixing</u> of the total energy.

If a gradient assumption is made for the large scale non-axisymmetric fluctuations, one could write

$$\overline{\rho'\, w_r'\, I''} \equiv \varepsilon \frac{\partial \bar{\bar{I}}^{(m)}}{\partial r} \qquad (2.21)$$

or

$$\overline{\rho'\, w_r'\, I''} \equiv \hat{\varepsilon} \frac{\partial \bar{\bar{I}}^{(m)}}{\partial n}$$

giving rise to a diflution type equation for the energy redistribution due to the non-axisymmetric flow field.

When the energy equation is written for the total energy of the averaged flow $\hat{I}^{(m)}$, the radial mixing term has to be added to the right hand side of equation (2.11), which becomes

$$\bar{w}_m^{(a)} \frac{\partial}{\partial m} \bar{\bar{I}}^{(m)} = \frac{1}{b r \bar{\rho}} \frac{\partial}{\partial n}(b\, r\, \overline{\rho'\, w_r'\, I''}) - \bar{w}_m^{(a)} \frac{\partial}{\partial m} \bar{\bar{k}}^{(m)} \qquad (2.22)$$

The non-constancy of the total energy of the averaged flow, $\hat{I}^{(a)}$ or $\hat{I}^{(m)}$, poses a fundamental problem in through-flow computations where the application of some form of total energy conservation has to be applied. As mentioned above, the calculations performed by Jennions and Stow (1985) in the case of gas turbine nozzle vanes, indicates that the kinetic energy terms in equations (2.2) or (2.11) are not negligible. One might wonder, therefore, what the influence this might have on the evaluation of the right-hand side terms in the radial equilibrium equation under the form of equation (1.33) for instance, Crocco's form.

The following argument tends to support the statement that the entropy variations, or more precisely the rotary stagnation pressure gradients, are more important than the enthalpy variations, at least for low speed flows.

From the isentropic relations between static and absolute or relative stagnation conditions, one has

$$T\,ds = dh - \frac{dp}{\rho}$$

or

$$T_0\,ds = dH - \frac{dp_0}{\rho_0}$$

where the subscript indicates absolute stagnation conditions. For rotary, relative conditions, whereby

$$T_0^* = T + \frac{\vec{w}^2}{2c_p} - \frac{\vec{u}^2}{2c_p} \qquad (2.23)$$

$$p_0^* = p\left(\frac{T_0^*}{T}\right)^{\gamma/\gamma-1}$$

one has

$$T_0^* \, ds = dI - \frac{dp_0^*}{\rho_0^*}$$

Hence

$$Tds - dI = \frac{T}{T_0^*} dI - \frac{T}{\rho_0^* T_0^*} dp_0^* - dI$$

$$= \left(\frac{T}{T_0^*} - 1\right) dI - \frac{rT}{p_0^*} dp_0^*$$

$$= \left(\frac{T}{T_0^*} - 1\right) dI - \frac{p}{\rho p_0^*} dp_0^* \qquad (2.24)$$

Introducing a rotary Mach number, M_0^*

$$M_0^{*2} = \frac{\vec{w}^2 - \vec{u}^2}{\gamma r T} \qquad (2.25)$$

the coefficients in equation (2.24) become,

$$\frac{T_0^* - T}{T_0^*} = 1 - \frac{1}{1 + \frac{\gamma-1}{2} M_0^{*2}} = \frac{\frac{\gamma-1}{2} M_0^{*2}}{1 + \frac{\gamma-1}{2} M_0^{*2}}$$

$$\frac{p}{p_0^*} = \left(\frac{1}{1 + \frac{\gamma-1}{2} M_0^{*2}}\right)^{\gamma/\gamma-1}$$

The terms $(T\vec{\nabla}s - \vec{\nabla}I)$ in the right-hand side of the momentum equation (1.33), becomes, independently of the type of averaged quantity considered

$$T\vec{\nabla}s - \vec{\nabla}I = \frac{\frac{\gamma-1}{2} M_0^{*2}}{1 + \frac{\gamma-1}{2} M_0^{*2}} \vec{\nabla}I - \frac{1}{\rho} \frac{1}{(1 + \frac{\gamma-1}{2} M_0^{*2})^{\gamma/\gamma-1}} \vec{\nabla}p_0^* \qquad (2.26)$$

In low speed compressors the blade exit rotary Mach number (2.25) should be small. This is in particular the case for the data of the UTRC experimental axial compressor runs discussed in the next section. When this is the case, the influence of the stagnation pressure variations is clearly dominating the through-flow and the momentum exchange.

One could therefore consider that small errors on the evaluation of the enthalpy transport will not affect significantly the radial equilibrium of the flow.

3. MOMENTUM AND MASS AVERAGED THROUGH-FLOW EQUATIONS

With density weighted, area averaged variables one can clearly define a consistent through-flow model where, next to the blade blockage factor b, interaction terms due to the non-axisymmetric secondary stresses $\overline{\overline{\tau}}^{(a)}$ are describing the three-dimensional effects on the average flow.

As mentioned in the introduction, mass averaged variables such as stagnation pressure and total enthalpy are more representative of the physical energy exchange than the corresponding area averaged variables. In an attempt to formulate a coherent through-flow model for mass-averaged variables we reconsider the passage averaged equations (1.20) to (1.22), where for simplicity the momentum equations are considered in the absolute frame of reference.

The mass conservation equation and the energy equation have already been discussed, with the following outcome.

Mass conservation

The natural averaged quantities are the density weighted, area averaged velocities which lead to the first equation (1.25), reproduced here

$$\frac{\partial}{\partial r}(\bar{\rho} b r \bar{w}_r^{(a)}) + \frac{\partial}{\partial z}(\bar{\rho} b r \bar{w}_z^{(a)}) = 0 \tag{3.1}$$

or in vector form

$$\vec{\nabla}(\bar{\rho} b r \bar{\vec{w}}^{(a)}) = 0 \tag{3.2}$$

3.1 Momentum Equations--Momentum Averaged Velocity Components

Considering equations (1.21) in the absolute system, it appears that it is not possible to define a unique momentum averaged velocity component, since different momentum flux components appear in the projections of the equation of motion. For instance, in the radial component, the averaged flux components $\overline{\rho v_r v_r}$, $\overline{\rho v_r w_z}$ and $\overline{\rho v_\theta v_\theta}$ occur, while in the axial projection one encounters the components $\overline{\rho v_z v_r}$ and $\overline{\rho v_z v_z}$. This leads to the definition of momentum averaged velocity components such as

$$\overline{\rho v_r v_r} = \overline{\rho v_r} \cdot \bar{\bar{v}}_r^{(r)} = \bar{\rho}\, \bar{\bar{v}}_r^{(r)} \bar{v}_r^{(a)} \tag{3.3}$$

$$\overline{\rho v_r v_r} = \frac{1}{b} \int_p^s \rho v_r\, v_r d\theta \tag{3.4}$$

and

$$\overline{\rho v_r v_z} = \overline{\rho v_r}\; \bar{\bar{v}}_z^{(r)} = \bar{\rho}\; \bar{v}_r^{(a)}\; \bar{\bar{v}}_z^{(r)} \tag{3.5}$$

$$\overline{\rho v_r v_z} = \frac{1}{b} \int_p^s \rho\, v_r\, v_z\, d\theta \tag{3.6}$$

One has also

$$\overline{\rho v_r v_z} = \bar{\rho}\; \bar{v}_z^{(a)}\; \bar{\bar{v}}_r^{(z)} \tag{3.7}$$

The averaged velocity component $\bar{\bar{v}}_r^{(r)}$ represents the average of v_r weighted by the radial momentum ρv_r. Similarly, from (3.6) and (3.7), one can define another averaged radial velocity component $\bar{\bar{v}}_r^{(z)}$, representing an average v_r, weighted this time by the axial momentum ρv_z. A priori there is no reason to consider these two components as equal. From the definition of the velocity fluctuations $\vec{v}''$, following equation (1.5), one can write for instance for $\overline{\rho v_r v_z}$,

$$\overline{\rho v_r v_z} = \bar{\rho}\ \bar{v}_r^{(a)}\ \bar{v}_z^{(a)} + \overline{\rho v_r'' v_z''} \qquad (3.8)$$

where $\overline{\rho v_r'' v_z''}$ are the components of the secondary stress tensor $\bar{\bar{\tau}}^{(S)}$ considered in the absolute system.

We define now six different blockage coefficients, K_{ij} where i and j represent the components (r, θ, z) by

$$K_{ij}^{-1} = \frac{\overline{\rho v_i v_j}}{\bar{\rho}\ \bar{v}_i^{(a)}\ \bar{v}_j^{(a)}} = \frac{\bar{\bar{v}}_j^{(i)}}{\bar{v}_j^{(a)}} \qquad (3.9)$$

Hence K_{ij}^{-1} is the ratio between the averaged velocity component v_j weighted with the momentum ρv_i divided by the corresponding density weighted, area averaged velocity component $\bar{v}_j^{(a)}$.

From the symmetry between i and j best seen in equation (3.10) one has also

$$K_{ij}^{-1} = \frac{\bar{\bar{v}}_i^{(j)}}{\bar{v}_i^{(a)}} = K_{ji}^{-1} \qquad (3.10)$$

or

$$K_{ij}\ \bar{\bar{v}}_i^{(j)} = \bar{v}_i^{(a)} \qquad (3.11)$$

This definition of a blockage coefficient with respect to a selected momentum component generalizes the definition of the aerodynamic blockage coefficient introduced in Dring (1984) and discussed more in details in Dring and Joslyn (1985). In these references, the averaged velocity $\overline{v}_z^{(m)}$ is determined from the mass averaged dynamic pressure, and mass averaged flow angles assuming constant flow angles equal to their averaged values.

From equation (3.9) one can also write

$$K_{ij}^{-1} = 1 + \frac{\overline{\rho v_i'' v_j''}}{\overline{\rho}\ \overline{v}_i^{(a)}\ \overline{v}_j^{(a)}} \qquad (3.12)$$

Introducing the $\overline{\overline{v}}_i^{(j)}$ component in the equations (1.21) leads to the following formulation for the radial component

$$\frac{1}{br}\frac{\partial}{\partial r}(\overline{\rho}\ \overline{v}_r^{(a)}\ \overline{\overline{v}}_r^{(r)} br) + \frac{1}{br}\frac{\partial}{\partial z}(\overline{\rho}\ \overline{v}_z^{(a)}\ \overline{\overline{v}}_r^{(z)} br) - \frac{\overline{\rho}\ \overline{v}_\theta^{(a)}\ \overline{\overline{v}}_\theta^{(\theta)}}{r}$$

$$= -\frac{\partial \overline{p}}{\partial r} + \overline{\rho}(F_{fr} + f_{Br}) \qquad (3.13)$$

or in function of the K_{ij} coefficients, eliminating the density, area averaged component,

$$\frac{1}{br}\frac{\partial}{\partial r}(\overline{\rho}\ K_{rr}\ \overline{\overline{v}}_r^{(r)}\overline{\overline{v}}_r^{(r)} br) + \frac{1}{br}\frac{\partial}{\partial z}(\overline{\rho}\ K_{rz}\ \overline{\overline{v}}_r^{(z)}\overline{\overline{v}}_z^{(r)} br)$$

$$- \frac{\overline{\rho}\ K_{\theta r}\ \overline{\overline{v}}_\theta^{(r)}\ \overline{\overline{v}}_\theta^{(\theta)}}{r} = -\frac{\partial \overline{p}}{\partial r} + \overline{\rho}(F_{fr} + f_{Br}) \qquad (3.14)$$

An alternative formulation is obtained as a function of the density weighted velocity components $\overline{v}_i^{(a)}$. Equation (3.13) becomes by eliminating the variables $\overline{\overline{v}}_i^{(j)}$,

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{v}_r^{(a)}\,\bar{v}_r^{(a)} brK_{rr}^{-1}) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{v}_z^{(a)}\,\bar{v}_r^{(a)} brK_{rz}^{-1})$$

$$- \frac{\rho\,\bar{v}_\theta^{(a)}\,\bar{v}_\theta^{(a)}}{r} K_{\theta\theta}^{-1} = -\frac{\partial\bar{p}}{\partial r} + \bar{\rho}(F_{fr} + f_{BR}) \qquad (3.15)$$

Equation (3.15) is to be compared to the projections of equation (1.15); for instance the radial component gives in the absolute system

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{v}_r^{(a)}\,\bar{v}_r^{(a)} br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{v}_z^{(a)}\,\bar{v}_r^{(a)} br) - \frac{\rho\bar{v}_\theta^{(a)}\bar{v}_\theta^{(a)}}{r}$$

$$= -\frac{\partial\bar{p}}{\partial r} + \bar{\rho}(F_{fr} + f_{Br}) - \frac{1}{br}\frac{\partial}{\partial r}(\overline{\rho v_r'' v_r''} br)$$

$$- \frac{1}{br}\frac{\partial}{\partial z}(\overline{\rho v_r'' v_z''} br) + \frac{\overline{\rho v_\theta'' v_\theta''}}{r} \qquad (3.16)$$

It can be seen from the relation (3.12) that both formulations are identical and that the coefficients K_{ij}^{-1} do contain the same information as the secondary stress $\bar{\bar{\tau}}^{(s)}$. Hence the interaction terms do not appear in equation (3.15), since the whole influence of the non-axisymmetry is contained in the K_{ij} coefficients. The other components of the momentum equation can be treated in a similar way.

Axial momentum equation

One obtains, from the second equation (1.21),

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{v}_z^{(a)}\,\bar{\bar{v}}_r^{(z)} br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{v}_z^{(a)}\,\bar{\bar{v}}_z^{(z)} br) = -\frac{\partial\bar{p}}{\partial z} + \bar{\rho}(F_{fz} + f_{Bz})$$

$$(3.17)$$

The alternative forms as a function of the momentum and density averaged velocity components are as follows.

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{\bar{v}}_z^{(r)}\,\bar{\bar{v}}_r^{(r)}K_{rr}br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{\bar{v}}_z^{(r)}\,\bar{\bar{v}}_z^{(z)}K_{rz}br)$$

$$= -\frac{\partial\bar{p}}{\partial z} + \bar{\rho}(F_{fz} + f_{Bz}) \qquad (3.18)$$

or

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{\bar{v}}_z^{(r)}\,\bar{\bar{v}}_r^{(z)}K_{zz}br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{\bar{v}}_z^{(z)}\,\bar{\bar{v}}_z^{(z)}K_{zz}br)$$

$$= -\frac{\partial\bar{p}}{\partial z} + \bar{\rho}(F_{fz} + f_{Bz}) \qquad (3.19)$$

In function of the $\bar{v}_j^{(a)}$ components one has

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{v}_z^{(a)}\,\bar{v}_r^{(a)}K_{rz}^{-1}br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{v}_z^{(a)}\,\bar{v}_z^{(a)}K_{zz}^{-1}br)$$

$$= -\frac{\partial\bar{p}}{\partial z} + \bar{\rho}(F_{fz} + f_{Bz}) \qquad (3.20)$$

where again, by comparison with equation (1.25) the interaction terms are fully absorbed by the K_{rz}^{-1} and K_{zz}^{-1} coefficients.

Tangential momentum equation

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{v}_r^{(a)}\,\bar{\bar{v}}_\theta^{(r)}br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{v}_z^{(a)}\,\bar{\bar{v}}_\theta^{(z)}br)$$

$$+ \frac{\bar{\rho}\bar{v}_\theta^{(a)}\bar{\bar{v}}_\theta^{(r)}}{r} = \bar{\rho}(F_{f\theta} + f_{B\theta}) \qquad (3.21)$$

Replacing the $\bar{v}^{(a)}$ components, leads to the following forms.

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{\bar{v}}_r^{(r)}\,\bar{\bar{v}}_\theta^{(r)}K_{rr}br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{\bar{v}}_z^{(r)}\,\bar{\bar{v}}_\theta^{(z)}K_{rz}br)$$

$$+ \frac{\rho\,\bar{\bar{v}}_r^{(\theta)}\,\bar{\bar{v}}_\theta^{(r)}}{r}K_{r\theta} = \bar{\rho}(F_{f\theta}+ f_{B\theta}) \qquad (3.22)$$

or introducing other K_{ij} variables

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{\bar{v}}_r^{(\theta)}\,\bar{\bar{v}}_\theta^{(z)}K_{z\theta}br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{\bar{v}}_z^{(\theta)}\,\bar{\bar{v}}^{(z)}K_{z\theta}br)$$

$$+ \frac{\rho\,\bar{\bar{v}}_r^{(\theta)}\,\bar{\bar{v}}_\theta^{(r)}}{r}K_{r\theta} = \bar{\rho}(F_{f\theta} + f_{B\theta}) \qquad (3.23)$$

In function of the density averaged velocities *one has*

$$\frac{1}{br}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{v}_r^{(a)}\,\bar{v}_\theta^{(a)}K_{r\theta}^{-1}br) + \frac{1}{br}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{v}_z^{(a)}\,\bar{v}_\theta^{(a)}K_{z\theta}^{-1}br)$$

$$+ \frac{\rho\,\bar{v}_r^{(a)}\,\bar{v}_\theta^{(a)}}{r}K_{r\theta}^{-1} = \bar{\rho}(F_{f\theta} + f_{B\theta}) \qquad (3.24)$$

to be compared with the third of the momentum equations (1.25).

Continuity equation

In a similar way, the continuity equation can be rewritten as a function of the momentum averaged velocity components, in different ways, according to the choice of the K_{ij} coefficients. From equations (3.1) and (3.11) one can write, with the absolute velocity component

$$\frac{\partial}{\partial r}(\bar{\rho}\,\bar{\bar{v}}_r^{(r)}\,K_{rr}br) + \frac{\partial}{\partial z}(\bar{\rho}\,\bar{\bar{v}}_z^{(r)}\,K_{zr}br) = 0 \tag{3.25}$$

or

$$\frac{\partial}{\partial r}(\bar{\rho}\,\bar{\bar{v}}_r^{(z)}\,K_{rz}br) + \frac{\partial}{\partial z}(\bar{\rho}\,\bar{\bar{v}}_z^{(z)}\,K_{zz}br) = 0 \tag{3.26}$$

A difficulty appears when the equations have to be derived in other forms, for instance in the non-conservative form or Crocco's form. Since one cannot introduce the continuity equation in the conservative form of the averaged momentum equation as a consequence of the non-equality of velocity components such as $\bar{\bar{v}}_r^{(r)} \neq \bar{\bar{v}}_r^{(z)}$.

Indeed, the left hand side of equation (3.13) can be worked out, leading to

$$\begin{aligned}\bar{\rho}\,\bar{v}_r^{(a)}\frac{\partial}{\partial r}\bar{\bar{v}}_r^{(r)} + \bar{\rho}\,\bar{v}_z^{(a)}\frac{\partial}{\partial z}\bar{\bar{v}}_r^{(z)} &+ \frac{1}{br}[\bar{\bar{v}}_r^{(r)}\frac{\partial}{\partial r}(\bar{\rho}\,\bar{v}_r^{(a)}br) \\ &+ \bar{\bar{v}}_r^{(z)}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{v}_z^{(a)}br)] - \frac{\bar{\rho}\,\bar{v}_\theta^{(a)}\,\bar{\bar{v}}_\theta^{(\theta)}}{r} \\ &= -\frac{\partial\bar{p}}{\partial r} + \bar{\rho}(F_{fr} + f_{Br})\end{aligned} \tag{3.27}$$

If $\bar{\bar{v}}_r^{(r)} = \bar{\bar{v}}_r^{(z)}$ one could factor out this velocity component, and the term in brackets would vanish due to mass conservation. Otherwise, one could not obtain Crocco's form in a consistent way, without the simultaneous appearance of different components $\overline{\overline{v}}_r^{(z)}$, $\overline{\overline{v}}_r^{(r)}$.

3.2 Simplifying Assumptions--Unique Blockage Coefficients

The assumption is made of a unique ratio between density and momentum averaged quantities. This corresponds to the assumption that

$$\begin{aligned}
\overline{\overline{v}}_r^{(z)} &= \overline{\overline{v}}_r^{(r)} = \overline{\overline{v}}_r^{(\theta)} \equiv \overline{\overline{v}}_r^{(m)} \\
\overline{\overline{v}}_z^{(z)} &= \overline{\overline{v}}_z^{(r)} = \overline{\overline{v}}_z^{(\theta)} \equiv \overline{\overline{v}}_z^{(m)} \\
\overline{\overline{v}}_\theta^{(z)} &= \overline{\overline{v}}_\theta^{(r)} = \overline{\overline{v}}_\theta^{(\theta)} \equiv \overline{\overline{v}}_\theta^{(m)}
\end{aligned} \tag{3.28}$$

implying that all the K_{ij} coefficients are equal. Hence, if this is satisfied, there is a unique definition of mass-averaged velocities, since for the axial component for instance,

$$\overline{\overline{v}}_z^{(m)} = \frac{\overline{\rho v_z v_z}}{\overline{\rho}\,\overline{v}_z^{(a)}} = \frac{\overline{\rho v_z v_m}}{\overline{\rho}\,\overline{v}_m^{(a)}} = K^{-1}\,\overline{v}_z^{(a)} \tag{3.29}$$

and similarly for the other components.

With this assumption, the continuity equation (3.25) or (3.26) becomes

$$\frac{\partial}{\partial r}(\overline{\rho}\,\overline{\overline{v}}_r^{(m)}\,Kbr) + \frac{\partial}{\partial z}(\overline{\rho}\,\overline{\overline{v}}_z^{(m)}\,Kbr) = 0 \tag{3.30}$$

This equation is similar to the density weighted formulation (1.25) where the replacement by mass-averaged velocities has led to the introduction of a global blockage coefficient, B = Kb. This coefficient is a product of the total blade blockage and of an aerodynamic blockage K due to the non-axisymmetry of the flow.

The momentum equations can be reduced as follows, considering first the radial component (3.14).

With the continuity equation (3.30), one obtains

$$\bar{\rho}K[\bar{\bar{v}}_r^{(m)} \frac{\partial}{\partial r} \bar{\bar{v}}_r^{(m)} + \bar{\bar{v}}_z^{(m)} \frac{\partial}{\partial z} \bar{\bar{v}}_r^{(m)}] - \frac{\rho \bar{\bar{v}}_\theta^{(m)} \bar{\bar{v}}_\theta^{(m)}}{r} K$$

$$= - \frac{\partial \bar{p}}{\partial r} + \bar{\rho}(F_{fr} + f_{Br}) \qquad (3.31)$$

Introducing equation (1.29) in the right hand side and writing

$$\frac{\rho \bar{\bar{v}}_\theta^{(m)\,2}}{r} = \frac{\bar{\bar{v}}_\theta^{(m)}}{r} \frac{\partial}{\partial r}(r\bar{\bar{v}}_\theta^{(m)}) - \bar{\bar{v}}_\theta^{(m)} \frac{\partial}{\partial r} \bar{\bar{v}}_\theta^{(m)}$$

one obtains, with the assumptions that the thermodynamic variables $\bar{T}^{(a)}$ and $\bar{h}^{(a)}$ relate to their mass averaged values in the same way as the velocity components,

$$\bar{\bar{h}}^{(m)} = K^{-1} \bar{h}^{(a)}$$

$$\bar{\bar{T}}^{(m)} = K^{-1} \bar{T}^{(a)}$$

$$\bar{\bar{v}}_r^{(m)} \frac{\partial}{\partial r} \bar{\bar{v}}_r^{(m)} + \bar{\bar{v}}_z^{(m)} \frac{\partial}{\partial z} \bar{\bar{v}}_r^{(m)} - \frac{\bar{\bar{v}}_\theta^{(m)}}{r} \frac{\partial}{\partial r}(r\bar{\bar{v}}_\theta^{(m)}) + \bar{\bar{v}}_\theta^{(m)} \frac{\partial}{\partial r} \bar{\bar{v}}_\theta^{(m)}$$

$$= \frac{T^{(a)}}{K} \frac{\partial s}{\partial r} - \frac{1}{K} \frac{\partial \bar{h}^{(a)}}{\partial r} + \frac{F_{fr}}{K} + \frac{f_{fr}}{K}$$

$$= \bar{\bar{T}}^{(m)} \frac{\partial s}{\partial r} - \frac{\partial}{\partial r} \bar{h}^{(a)}/K + \frac{F_{fr} + f_{fr}}{K} + \bar{h}^{(a)} \frac{\partial K^{-1}}{\partial r}$$

$$= \bar{\bar{T}}^{(m)} \frac{\partial s}{\partial r} - \frac{\partial}{\partial r}[\bar{\bar{h}}^{(m)} + \frac{(\vec{\bar{\bar{v}}}^{(m)})^2}{2}] + \frac{1}{2} \frac{\partial}{\partial r}(\bar{\bar{v}}^{(m)})^2$$

$$+ \frac{F_{fr} + f_{Br}}{K} + \bar{h}^{(a)} \frac{\partial K^{-1}}{\partial r} \qquad (3.32)$$

Finally one obtains

$$\bar{\bar{v}}_z^{(m)} [\frac{\partial}{\partial z} \bar{\bar{v}}_r^{(m)} - \frac{\partial}{\partial r} \bar{\bar{v}}_z^{(m)}] - \frac{\bar{\bar{v}}_\theta^{(m)}}{r} \frac{\partial}{\partial r}(r\bar{\bar{v}}_\theta^{(m)})$$

$$= \bar{\bar{T}}^{(m)} \frac{\partial s}{\partial r} - \frac{\partial}{\partial r} \hat{H}^{(m)} + \frac{F_{fr} + f_{Br}}{K} - \bar{\bar{h}}^{(m)} \frac{\partial}{\partial r} \ln K \qquad (3.33)$$

Within the above simplifying assumptions, the through-flow equations can be interpreted as functions of the mass-averaged flow variables. The influence of the "interaction" terms in equation (1.25), is taken up by the K-coefficients.

In the continuity equation the geometrical blockage factor $b = 1 - d/s$ is to be multiplied by K, leading to an overall blockage coefficient

$$B = bK$$

which has to be taken into account also in the axial regions between adjacent blade rows, e.g., where $b = 1$, but $K \neq 1$.

The additional term $[\bar{h}^{(m)} \frac{\partial}{\partial r} \ln K]$ can be added to the force terms $[F_{fr} + f_{Br}]$ divided by K and form a generalized force component. Remember that f_{Br} also contains a contribution equal to $\bar{p} \frac{\partial}{\partial r} \ln b$.

Equations (3.30) and (3.33) are consistent with the radial equilibrium treatments of Calvert and Ginder (1985) as well as Dring and Joslyn (1985), in terms of mass-averaged quantities and a simple aerodynamic blockage coefficient replacing the interaction terms in the density weighted formulation.

The energy equation follows from (2.22) or

$$\bar{\bar{w}}_m^{(m)} \frac{\partial}{\partial m} \hat{I}^{(m)} = - \bar{\bar{w}}_m^{(m)} \frac{\partial}{\partial m} \bar{\bar{k}}^{(m)} + \frac{1}{\bar{\rho} br} \frac{\partial}{\partial h} (br \overline{\rho' w_r' I''}) \tag{3.34}$$

In a non-rotating blade row, the above equation reduces to

$$\bar{\bar{w}}_m^{(m)} \frac{\partial}{\partial m} \hat{H}^{(m)} = - \bar{\bar{w}}_m^{(m)} \frac{\partial}{\partial m} \bar{\bar{k}}^{(m)} + \frac{1}{\bar{\rho} br} \frac{\partial}{\partial h} (br \overline{\rho' w_r' H''}) \tag{3.35}$$

showing that $\hat{H}^{(m)}$ as appearing in equation (3.33) is not conserved, even in a non-rotating blade row. However as discussed in section 2, in combination with the entropy term, the dominating effects will most probably come from the stagnation pressure variations.

Note that the tangential equation (3.22) becomes

$$\frac{\bar{\rho}\,\bar{\bar{w}}_m^{(m)}}{r}\frac{\partial}{\partial m}[r\bar{\bar{v}}_\theta^{(m)}] = \bar{\rho}K(F_{f\theta} + f_{B\theta}) \tag{3.36}$$

Comparison With Data

From the data base collected in the last years at UTRC, the K_{ij} coefficients can be evaluated at different stations of a single and two-stage axial compressors.

Figures 3.1 to 3.3 show the difference between mass-averaged and area-averaged static and absolute and relative stagnation pressures at the exits of stator 1, rotor 2 and stator 2 of the two-stage compressor reported in Dring et al. (1982), (1983).

The common observation is the nearly axisymmetric behavior of the static pressure, while the stagnation pressures show large non-axisymmetric effects in the end-wall regions.

Figures 3.4 to 3.6 show the corresponding ratios of mass to area averaged absolute or relative velocities squared, as well as the ratios of the averaged kinetic energies. The curves follow very closely the stagnation pressure variations, as expected since static pressure is nearly constant. In addition, the difference between the two curves is a measure of the influence of the kinetic energy of the fluctuations, $\bar{\bar{k}}^{(m)}$. This influence is mostly sensible in the end wall regions as can be seen from figure 3.7 where the quantity $[1 + \bar{\bar{k}}^{(m)}/\bar{w}^{(m)2}]$

is plotted in function of space. Figures 3.8 to 3.10 display the K_{ij} coefficients at the same three locations.

The validity of the uniformity assumption (3.28) can be estimated on the basis of these results. For the components not involving the radial velocity, the assumption of equality of the K_{ij} is satisfied with an acceptable accuracy even in the end wall regions.

The results of the coefficients involving the radial velocity components are more puzzling. It should be noticed however that due to their small magnitude, a large error is connected to the determination of the K_{ri} coefficients, and more data would be required from turbomachines having larger average radial velocities.

4. ESTIMATION OF BLOCKAGE FACTOR

If the blade to blade flow can be separated into a wake and an inviscid region, one can easily derive a relation between the unique blockage factor and boundary layer or wake parameters, implying a connection with total pressure loss coefficients.

In figure 4.1 from Dring and Joslyn (1985), the above assumption is valid for the flow at exit of the rotor at all span locations, with the exception of the tip clearance flow which completely perturbs the blade to blade distribution. This is confirmed by the single and 2-stage data.

If β_2 is the outlet flow angle, the boundary layer thickness is measured in the direction normal to the flow, that is a displacement thickness δ^* is defined by, see figure 4.2

$$\delta^* = \int_p^s \left(1 - \frac{\rho v}{\rho_e v_e}\right) dn = \int_p^s \left(1 - \frac{\rho v}{\rho_e v_e}\right) dy \cos\beta_2 \qquad (4.1)$$

or

$$\frac{\delta^*}{bs\cos\beta_2} = \frac{1}{sb}\int_p^s \left(1 - \frac{\rho v}{\rho_e v_e}\right) dy \equiv \frac{\delta_p^* + \delta_s^* + t_{TE}}{bs\cos\beta_2} \qquad (4.2)$$

Note that this thickness δ^* contains the contribution both from the section and pressure surfaces, δ_p^*, δ_s^* and the trailing edge thickness t_{TE}.

Similarly, a momentum thickness θ, is defined by

$$\frac{\theta}{s\cos\beta_2} = \frac{1}{sb}\int_p^s \left(1 - \frac{v}{v_e}\right) \frac{\rho v}{\rho_e v_e}\, dy \qquad (4.3)$$

The density averaged absolute velocity is related to δ^* by

$$\bar{\rho}\,\bar{v}^{(a)} = \frac{1}{sb}\int_p^s \rho\, v dy = \rho_e\, v_e \left(1 - \frac{\delta^*}{bs\cos\beta_2}\right) \qquad (4.4)$$

If ρ_e is taken equal to the averaged density

$$\rho_e = \bar{\rho} \qquad (4.5)$$

and defining a dimensionless displacement thickness Δ_1 by

$$\Delta_1 = \frac{\delta^*}{bs\cos\beta_2} = \frac{\delta_p^* + \delta_s^* + t_{TE}}{bs\cos\beta_2} \qquad (4.6)$$

one has

$$\bar{v}^{(a)} = v_e(s - \Delta_1) \qquad (4.7)$$

The mass-averaged velocity, can be estimated as follows

$$\bar{\rho}\,\bar{v}_m^{(a)}\bar{v}^{(m)} = \frac{1}{bs}\int_p^s \rho\, v_m v\, dy \qquad (4.8)$$

or, assuming a uniform flow angle in agreement with the hypothesis of a unique blockage coefficient K,

$$\bar{\rho}\,\bar{v}^{(a)}\,\bar{v}^{(m)} = \frac{1}{bs}\int_p^s \rho\, v^2\, dy \qquad (4.9)$$

Defining a dimensionless momentum thickness for the suction plus pressure side boundary layer

$$\Delta_2 = \frac{\theta}{bs\cos\beta_2} \equiv \frac{\theta_p + \theta_s}{bs\cos\beta_2} \qquad (4.10)$$

one obtains

$$\frac{\bar{v}^{(m)}}{v_e} = \frac{1-\Delta_1-\Delta_2}{1-\Delta_1} = 1 - \frac{\Delta_2}{1-\Delta_1} \qquad (4.11)$$

The blockage factor K

$$K = \frac{\bar{v}^{(a)}}{\bar{v}^{(m)}} \qquad (4.12)$$

is therefore completely defined by the boundary layer parameters

$$K = \frac{1-\Delta_1}{1-\dfrac{\Delta_2}{1-\Delta_1}} \simeq 1 - \Delta_1 + \Delta_2 \qquad (4.13)$$

This relation is an alternative form for equation (4) of Dring (1984) relating the aerodynamic blockage coefficient to the total pressure loss coefficient $\frac{\Delta p_t}{q}$ where q is an appropriate dynamic head. When q is taken as the downstream dynamic head in the mixed-out flow, then

$$\frac{\Delta p_t}{q} \cong 2 \frac{\Delta_2}{1 - \Delta_1} \tag{4.14}$$

and

$$K = \frac{1 - \Delta_1}{1 - \frac{1}{2}\left(\frac{\Delta p_t}{q}\right)} \simeq \frac{1 - \Delta_1}{\sqrt{1 - (\Delta p_t/q)}} \tag{4.15}$$

which is the relation derived in Dring (1984).

These relations should allow the designer to estimate the blockage factor in relation with the assumed loss distribution.

Conclusions

A detailed analysis of the momentum averaged formulation has shown that a simplified model can be obtained, in the line of more intuitive considerations, if a simple blockage coefficient can be assumed.

This coefficient is then defined as the ratio between the density weighted, area averaged velocity and the mass averaged velocity. It contains the full information of the influence of the non-axisymmetry of the real flow on its averaged through-flow.

The present analysis therefore confirms the importance of the aerodynamic blockage as a major parameter to be introduced to the through-flow design systems, instead of the interaction terms based on the secondary stresses.

When the flow can be separated into a wake region and an inviscid zone, a relation can be established between this blockage coefficient and wake parameters or loss coefficients. This is however not possible when strong leakage and strong end-wall stall is present and no wake regions can be defined.

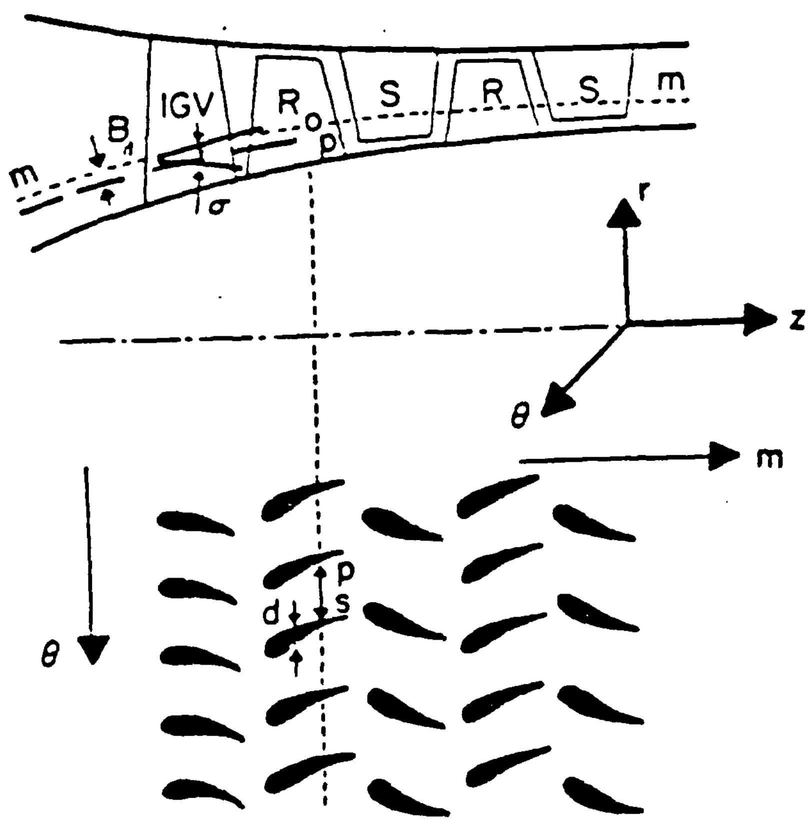

Figure 1.1 - Through-flow modeling - meridional and blade-to-blade planes.

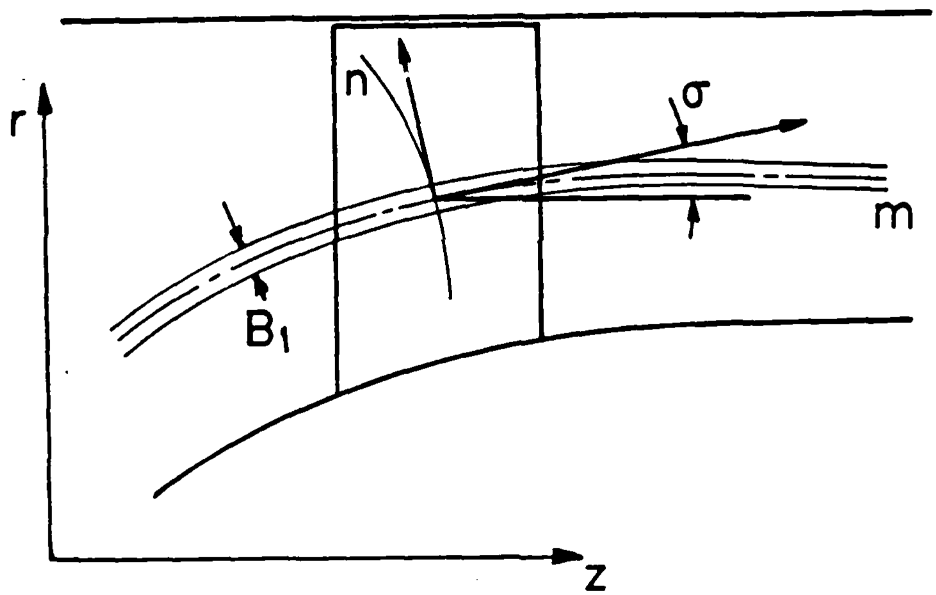

Figure 1.2 Trace of axisymmetric streamsurface in a meridional plane section.

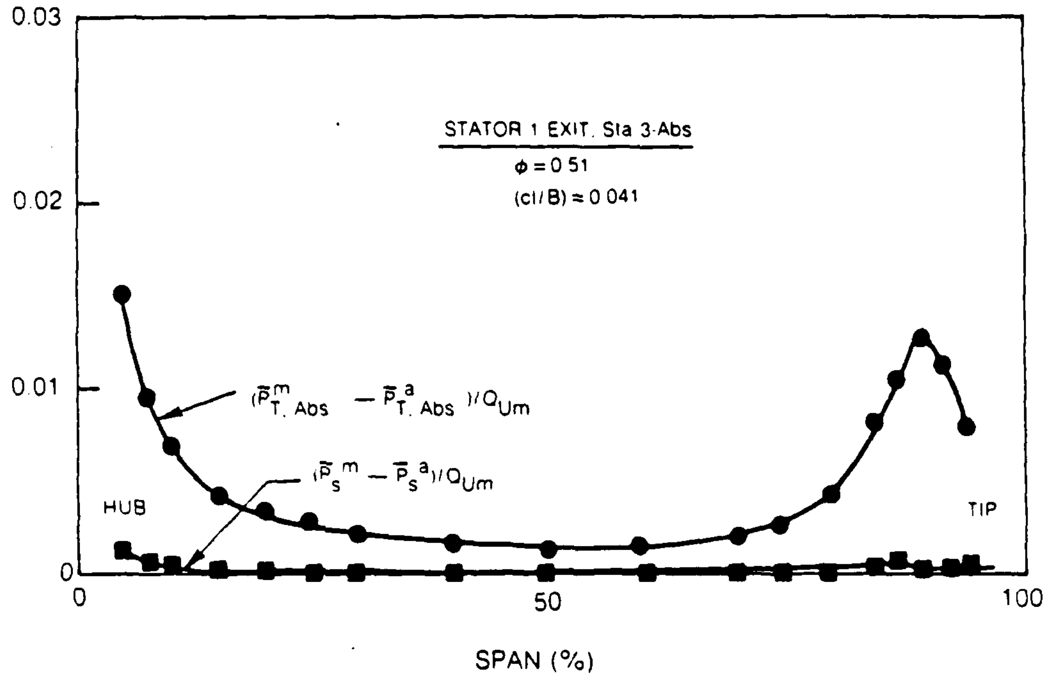

Figure 3.1 - Distribution of the difference between mass-averaged and area averaged static and total pressures at stator 1 exit.

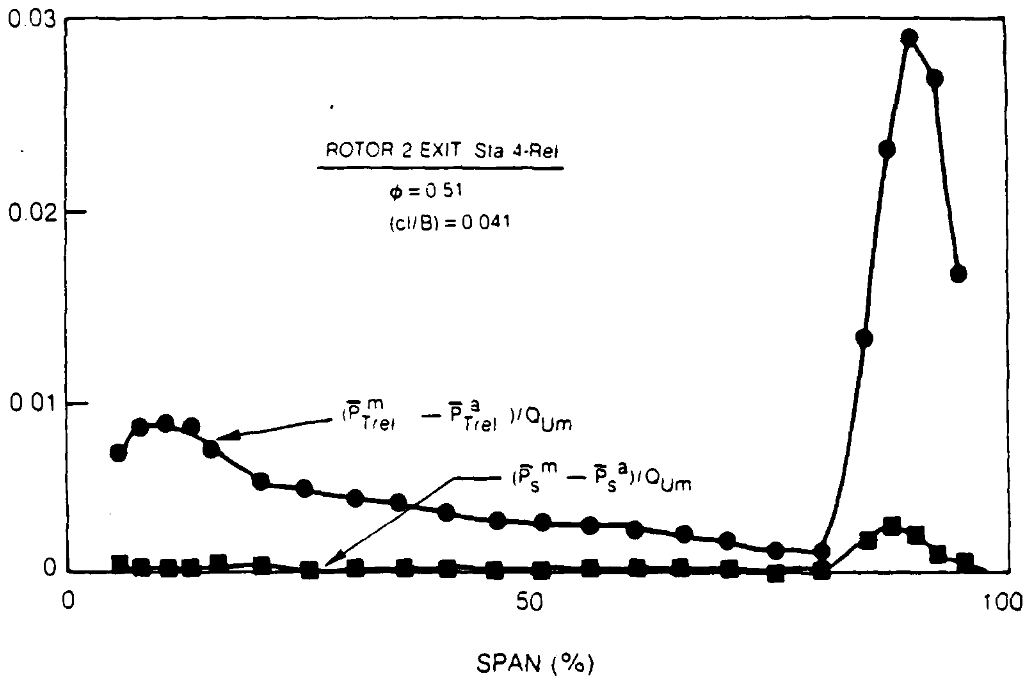

Figure 3.2 - Distribution of the difference between mass-averaged and area averaged static and total pressures at rotor 2 exit.

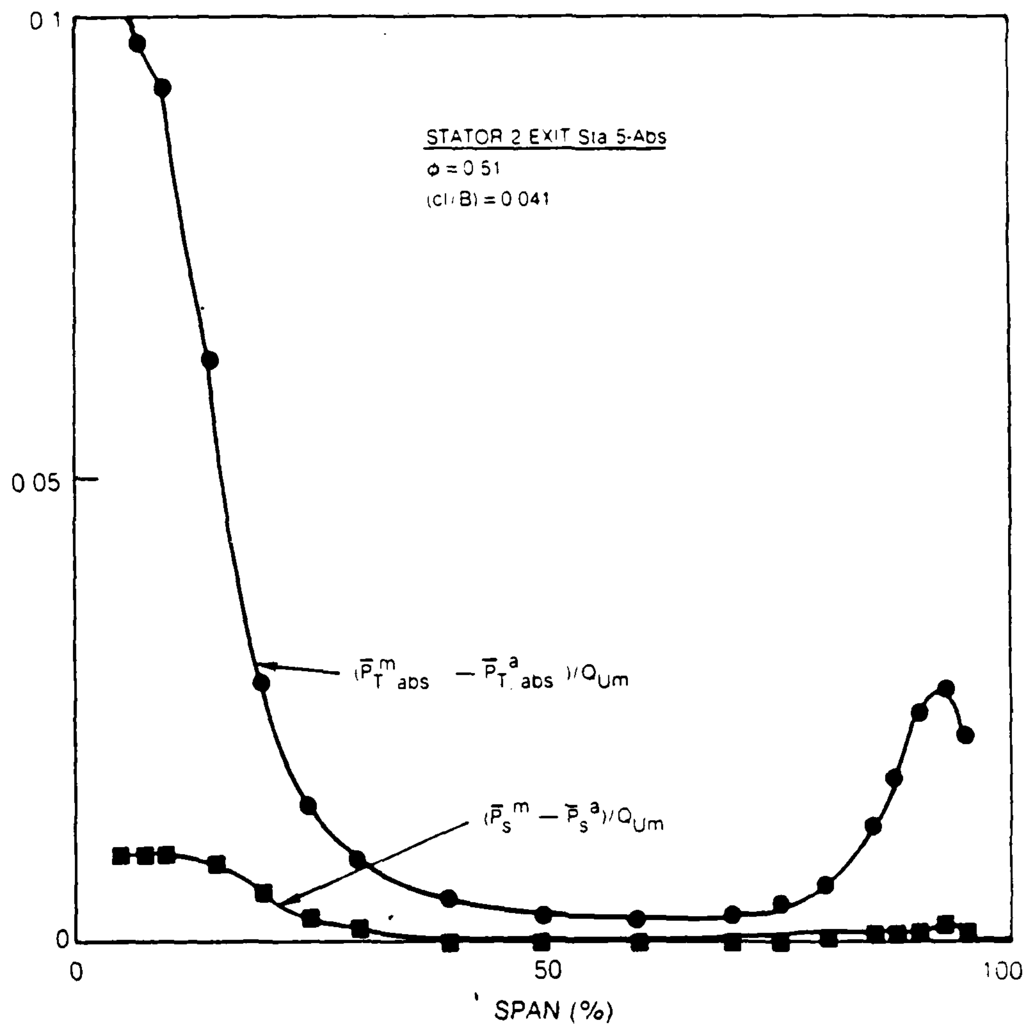

Figure 3.3 - Distribution of the difference between mass-averaged and area averaged static and total pressures at stator 2 exit.

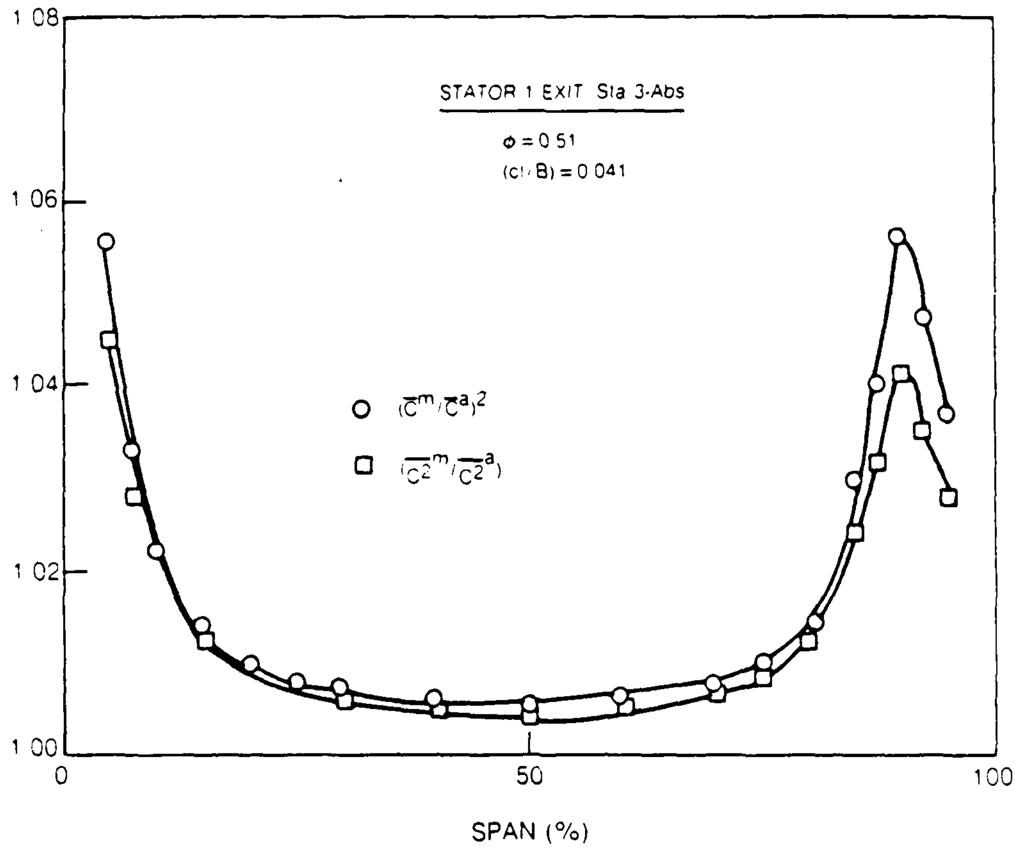

Figure 3.4 - Ratio of mass-averaged and area averaged absolute velocities at stator 1 exit.

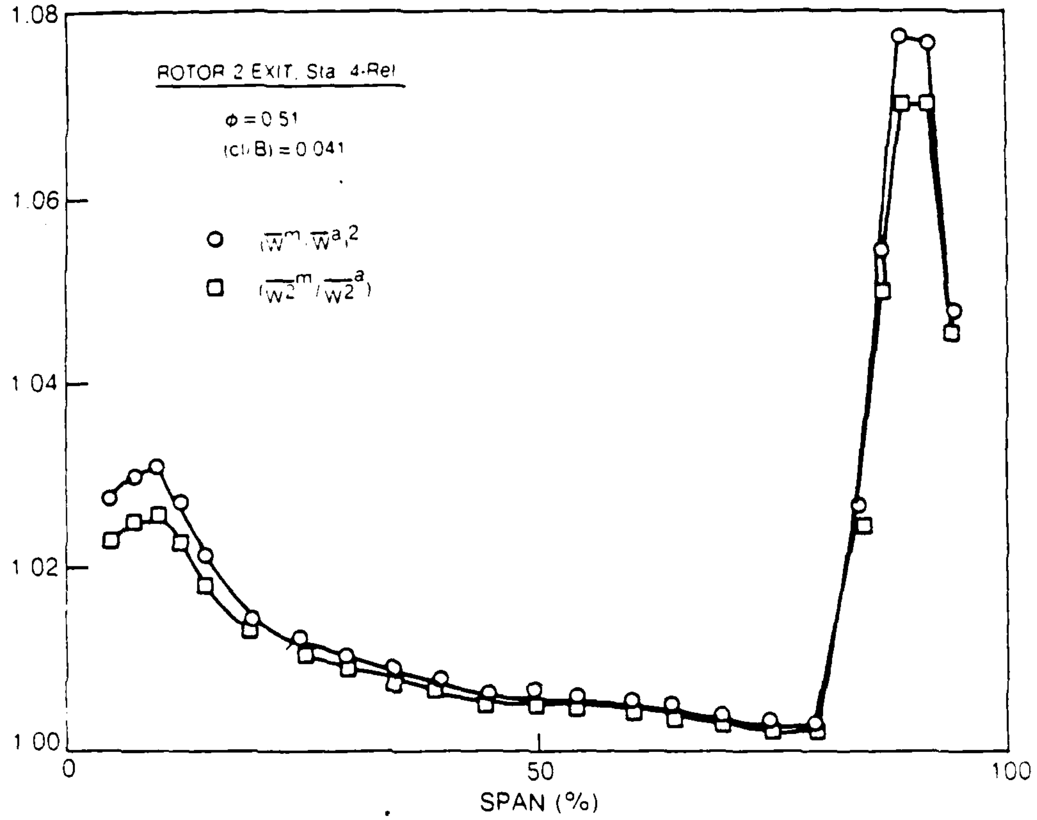

Figure 3.5 - Ratio of mass-averaged and area averaged relative velocities at rotor 2 exit.

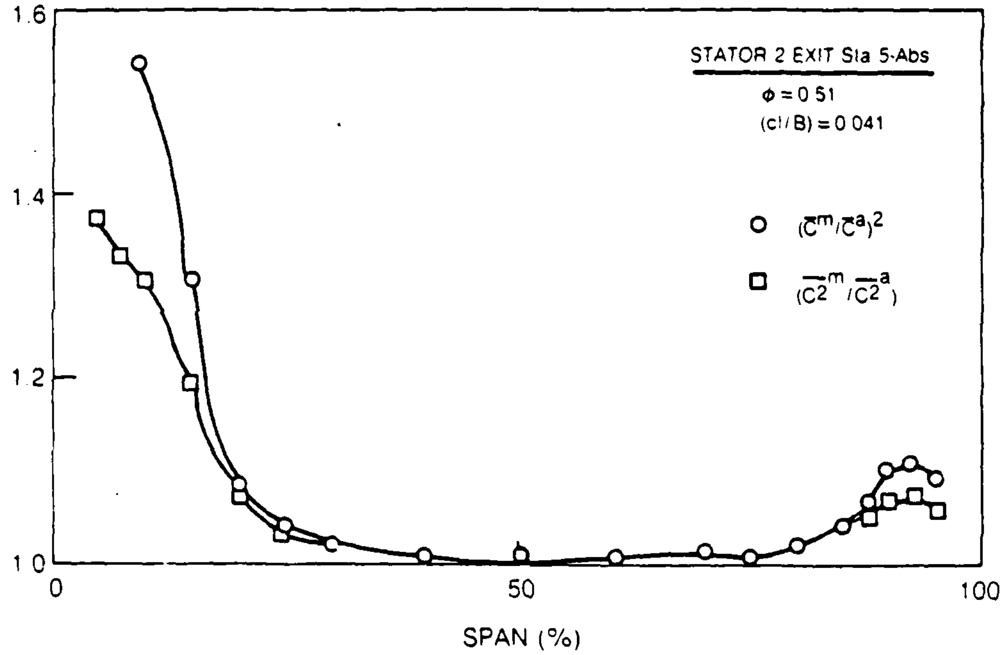

Figure 3.6 - Ratio of mass-averaged and area averaged absolute velocities at stator 2 exit.

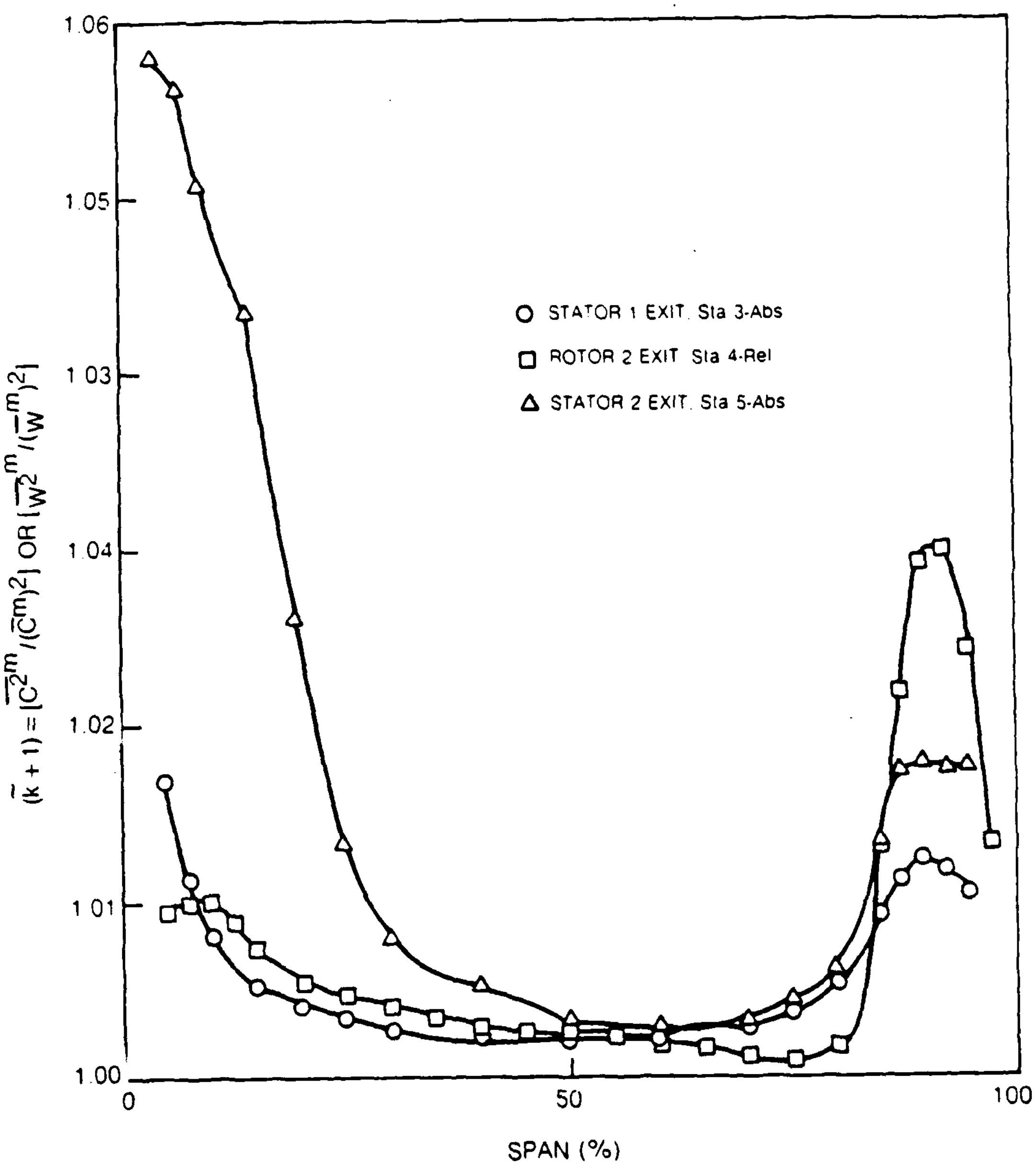

Figure 3.7 - Comparison of ratio of mass-averaged and area averaged at stator 1, rotor 2 and stator 2 exits.

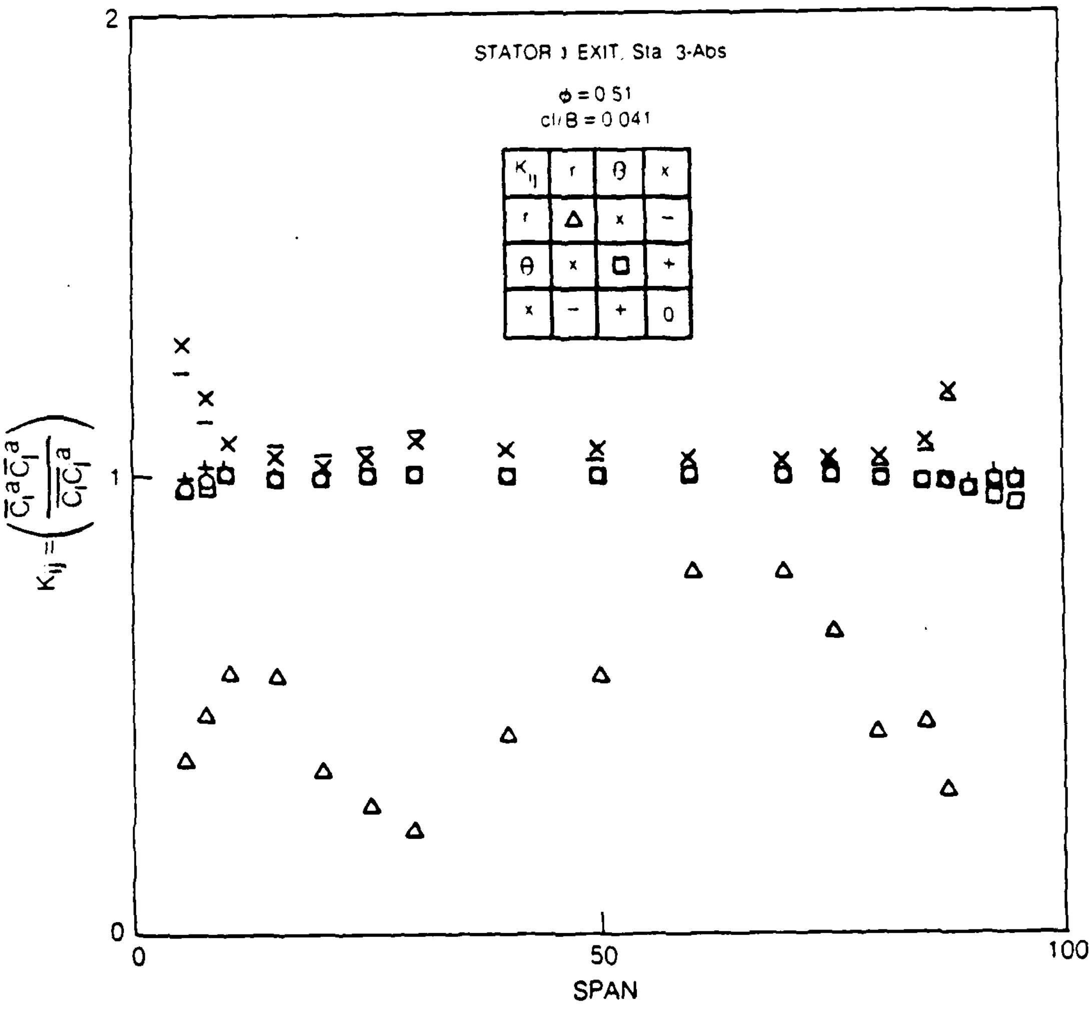

Figure 3.8 - Distribution of K_{ij} coefficients at first stator exit.

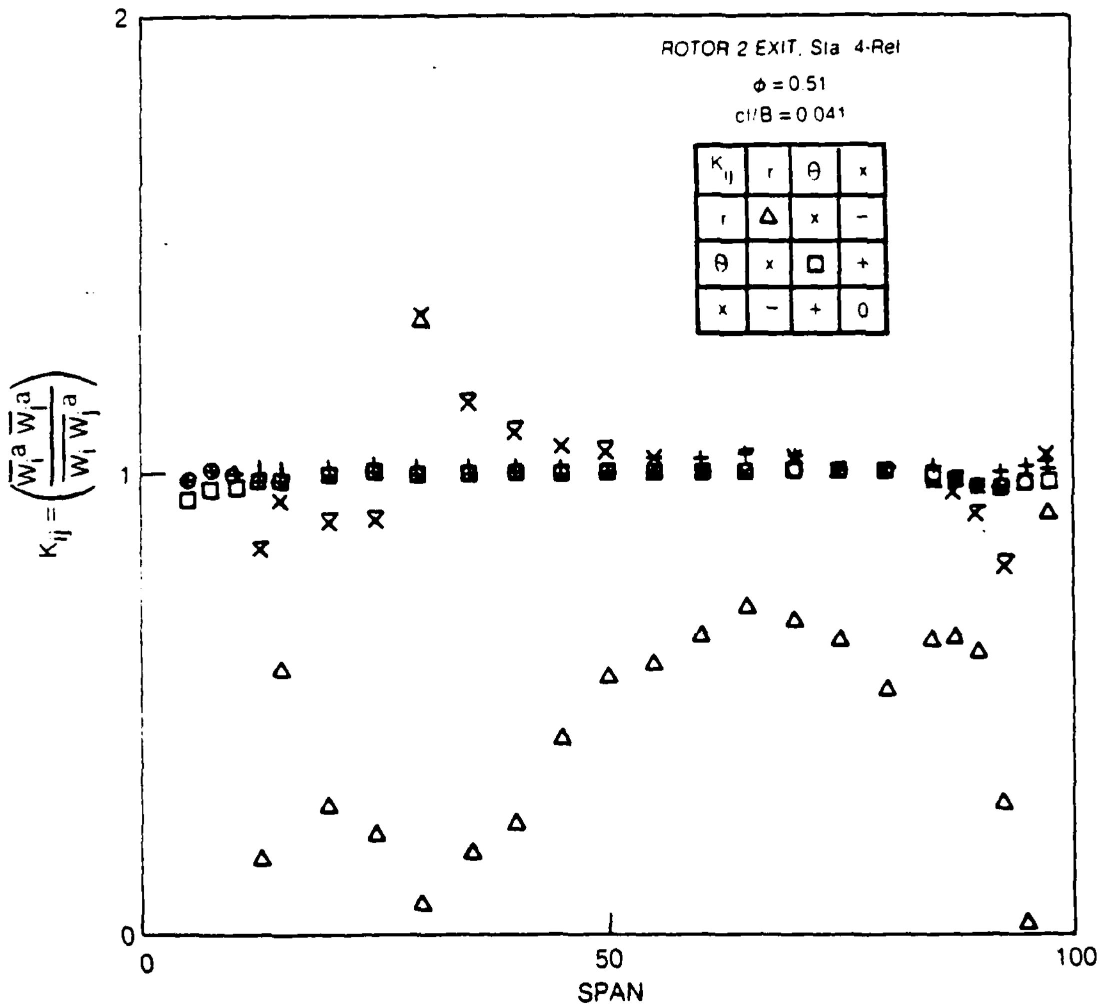

Figure 3.9 - Distribution of K_{ij} coefficients at rotor 2 exit.

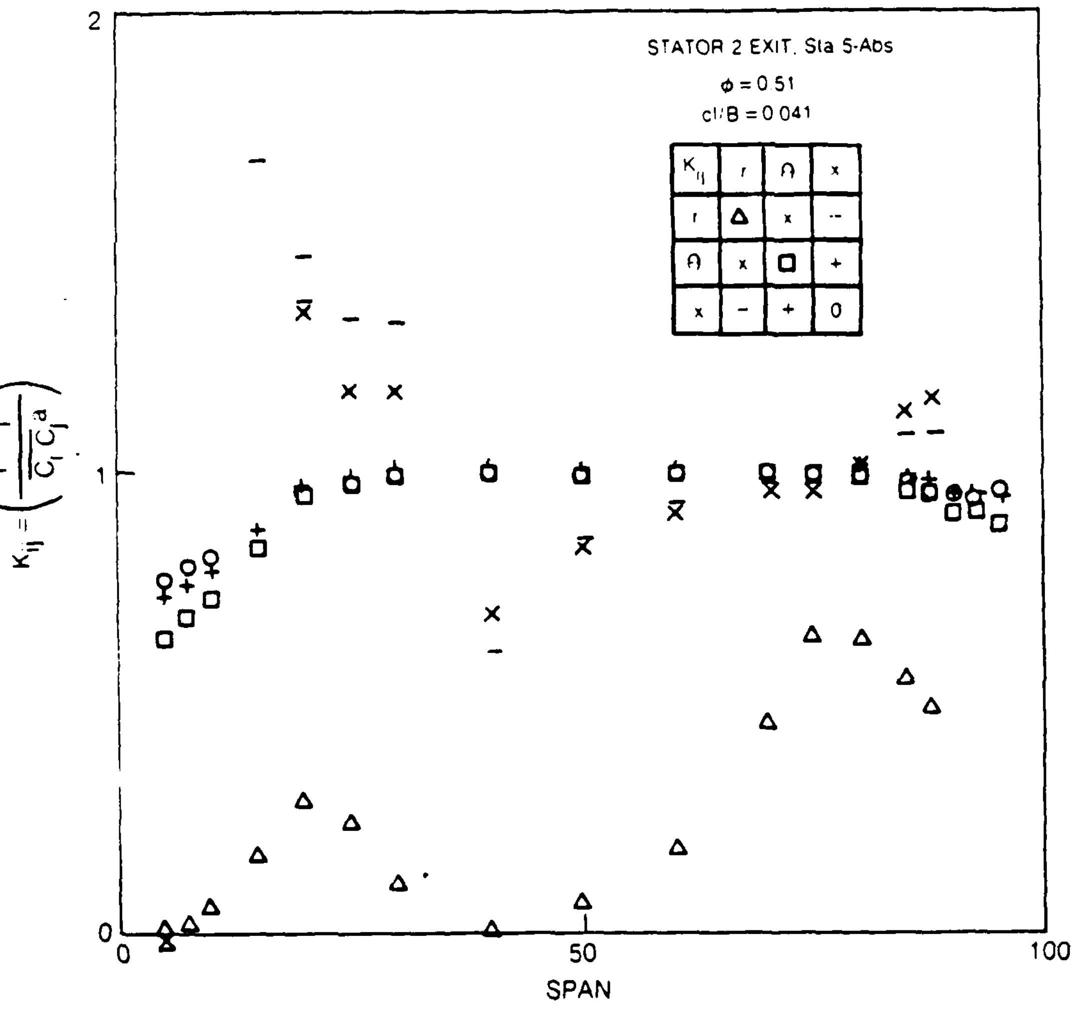

Figure 3.10 - Distribution of K_{ij} coefficients at stator 2 exit.

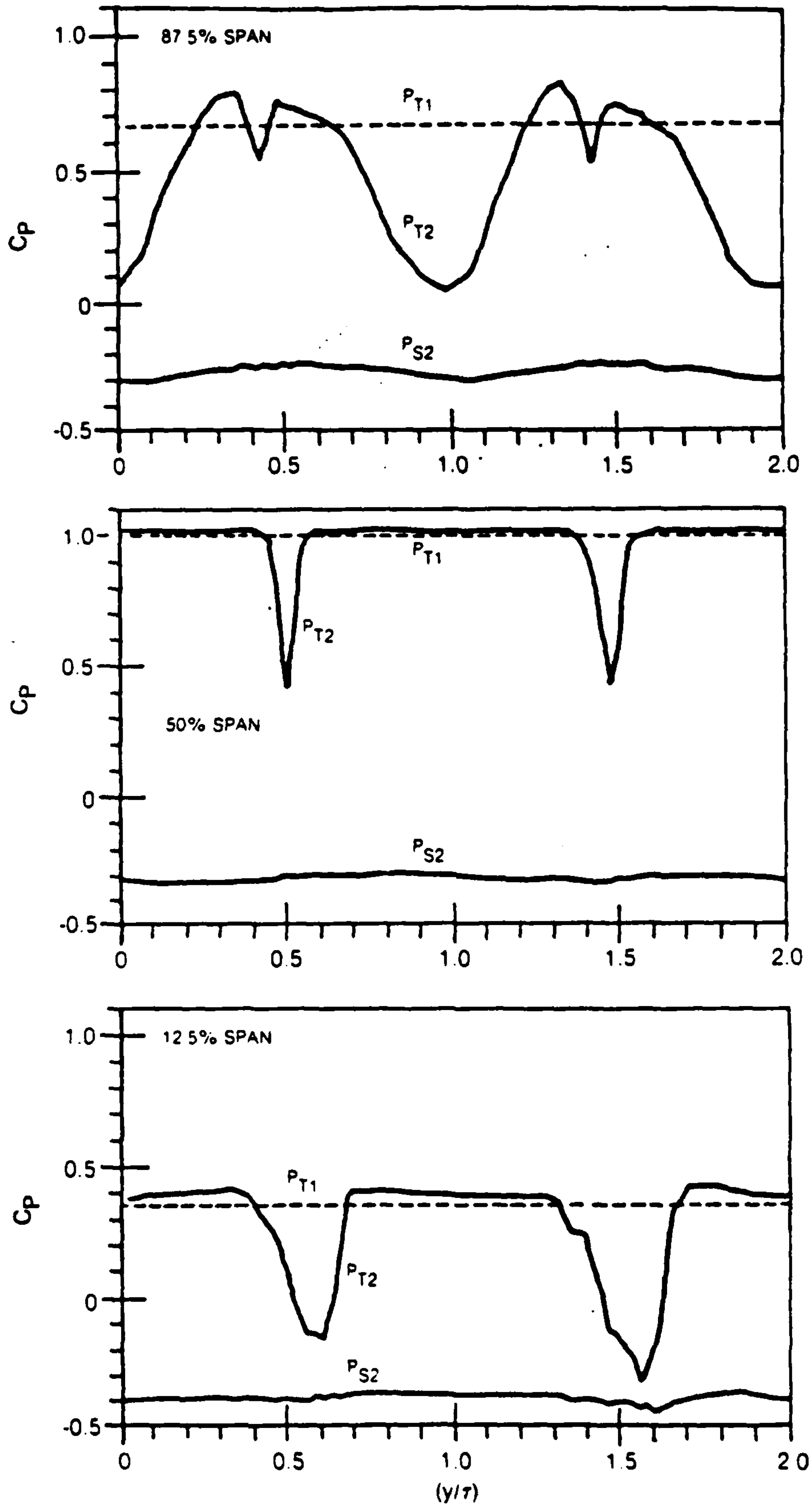

Figure 4.1 - Single-stage rotor wake profiles, thick inlet boundary layers Ø = 0.85, 30% aft.

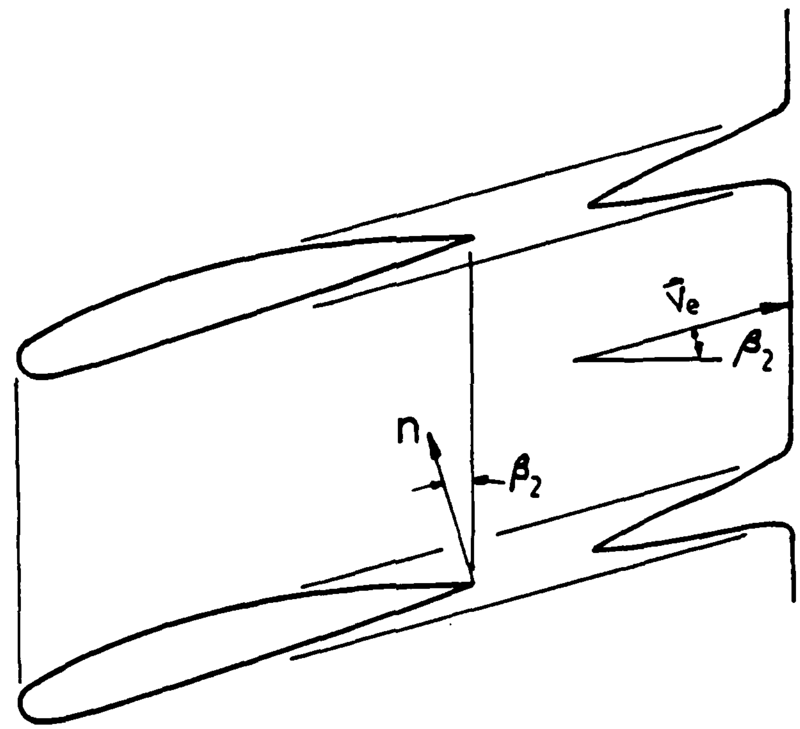

Figure 4.2 - Wake and inviscid flow regions

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